**Interactions**

Agents perform actions, and in a society those actions are relevant to other agents. Interaction amongst agents is, at the very least, one of the fundamental media of social cause and effect. We distinguish interactions as those actions which result in a modification of the subsequent action of the other participant(s) in the interaction. Given that the action of an agent is described by the action function *Ax*(*qx*, *cx*), where *Ax* stands for whatever function it is decided best suits the purpose, *qx* describes the quality of *x* at the time of the action and *cx* describes the context of *x* at the time of the action, it’s clear that there are (at least) two sites in which we can locate the possible cause of this effect.

***Act Effects***

1. *Context Effects*

Consider the effects produced by modification of contexts

Let *x* and *y* be two agents,

*ax,i = Ax*(*qx,i*, *c*x*,i*) is the action that *x* takes in the initial context *c*x,*i* of *x*, and

*C*(*ax,i*, *cy*,*i*) is the effect on the context of *y* of the action that *x* takes; then

we say that *ax,i* has an Act Effect Through Context on *y* and write *eC*(*ax,i*, *y*) iff:

(*ay*  *y*)[*Prob*(*Ay*(*qy,i*, *cy,i*)=*ay*)  *Prob*(*Ay*(*qy,i*, *C*(*ax,i*, *cy*,*i*))=*ay*)]

» *The probabilities of some particular actions being produced by y are altered by the change in its context produced by the action of x.*

* Subscripts in *i*, *j*, … are time markers (for time supposed to be in discrete steps.)
* Assume that the act effect continues into all subsequent times unless actually annulled
* The definition is stated in terms of probabilities because uncertainties in arguments or in the action function itself (in any practically plausible version) mean that we think in terms of classes of actions that are more or less likely to be produced in the contexts as we know them given the quality as we know it of the agent. However, if an appeal to probabilities is held to be objectionable for whatever reason (perhaps because it tends to make the definition a *post facto* classification, or it suggests an operationalization of the concept, or it externalises the definition, or it suggests that the action function is probabilistic rather than deterministic, or etc.) one could simply restate it without any inappropriate consequences as

*Ay*(*qy,i*, *cy,i*)  *Ay*(*qy,i*, *C*(*ax,i*, *cy*,*i*))

1. *Quality Effects*

Consider now the effects produced by modification of qualities

Let *x* and *y* be two agents,

*ax,i = Ax*(*qx,i*, *c*x*,i*) is the action that *x* takes in the initial context *c*x,*i* of *x*, and

*Q*(*ax,i*, *qy*,*i*) is the effect on the quality of *y* of the action that *x* takes; then

we say that *ax,i* has an Act Effect Through Quality on *y* and write *eQ*(*ax,i*, *y*) iff:

(*ay*  *y*)[*Prob*(*Ay*(*qy,i*, *cy,i*)=*ay*)  *Prob*(*Ay*(*Q*(*ax,i*, *qy*,*i*), *cy,i*)=*ay*)]

» *The probabilities of some particular actions being produced by y are altered by the change in its quality produced by the action of x.*

* Subscripts in *i*, *j*, … are time markers (for time supposed to be in discrete steps.)
* Assume that the act effect continues into all subsequent times unless actually annulled
* As for the context case, a preferable form might be

*Ay*(*qy,i*, *cy,i*)  *Ay*(*Q*(*ax,i*, *qy*,*i*), *cy,i*)

1. *Combined Effects*

The general case allows for effects both through quality and context.

Let *x* and *y* be two agents,

*ax,i = Ax*(*qx,i*, *c*x*,i*) is the action that *x* takes in the initial context *c*x,*i* of *x*, and

*C*(*ax,i*, *cy*,*i*) is the effect on the context of *y* of the action that *x* takes, and

*Q*(*ax,i*, *qy*,*i*) is the effect on the quality of *y* of the action that *x* takes; then

we say that *ax,i* has an Act Effect on *y* and write *e*(*ax,i*, *y*) iff:

(*ay*  *y*)[*Prob*(*Ay*(*qy,i*, *cy,i*)=*ay*)  *Prob*(*Ay*(*Q*(*ax,i*, *qy*,*i*), *C*(*ax,i*, *cy*,*i*))=*ay*)]

* Subscripts in *i*, *j*, … are time markers (for time supposed to be in discrete steps.)
* Assume that the act effect continues into all subsequent times unless actually annulled
* As before, a preferable form might be

*Ay*(*qy,i*, *cy,i*)  *Ay*(*Q*(*ax,i*, *qy*,*i*), *C*(*ax,i*, *cy*,*i*))

***Interactions***

Let *S* *X,*Let *JS* = (*a1*, …, *an*) be a sequence of actions by the members of *S.*We call *JS* an Interaction in *S* if:

1. [Relevance] (*ai*  *JS*)[ *ai* *x* *y**x*  *S*)[*e*(*ax,i*, *y*)]]

» *Every action by an agent in the sequence has an act effect on another agent in S*

1. [Participation] (*x*  *S*)*ai*  *JS*)[ *ai* *x*]

» *Every agent in S performs some action in the sequence*

1. [Completeness] (*x*  *S*)(*y**x*  *S*)(*i*)[ *eQ*(*ay,i*, *x*) ]

» *Every agent in S is subject to an act effect by another agent in S*

* Other conditions may be added for whatever reason: the point is to include as many as possible of those things that are typically thought of as an interaction, without including too many that are not thought of as interactions.

We can diagram actions and their effects conveniently on an Action Table. This is particularly useful for mapping interactions. For example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *i* = 0 | *x1* | *x2* | *x3* | *x4* |
| 1 | ***a*** |  | *c* | *c* |
| 2 |  | *q,**c* | ***a*** |  |
| 3 | *q* | ***a*** | *c* |  |
| 4 | *c* |  | ***a*** | *q,**c* |
| 5 | ***a*** | *c* | *c* | *c* |
| 6 | *q,**c* |  | *q,**c* | ***a*** |

Where *q,**c* in any cell indicates the change from the state that holds in the cell above. We understand the top of each column as implying the initial states for each agent. The columns we call Agent Histories and the rows we call Event Levels. Cell contents are Interaction Events.

From the conditions for an interaction described above we make the following observations

1. Every event level contains an action and a state change
2. Every agent history contains an action and a state change
3. The final state of each agent can be found by combining the state changes in its history with its initial state (not marked)

In the table just given we note that the sub-table for *x1*, *x2*, *x3* at *i* = 0, 1, 2, also describes an interaction. This is a general possibility that we need to be able to describe. Thus:

If *JS* = (*a1*, …, *an*) is an interaction and (*j*, *m*]  [1, *n*])(*S1*  *S*)[*J1,S1* = (*aj*, …, *am*) is an interaction on *S1*] then *J1,S1* is a Subsidiary Interaction of *JS*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Example:**   |  |  |  |  | | --- | --- | --- | --- | |  | *Alan* | *Bob* | *Claire* | | 1 | ***a*: Enter room** | *q*: Recognise Alan’s presence | *q*: Recognise the presence of an unknown person | | 2 | *q*: Acknowledge Bob’s greeting | ***a*: Greet and offer hand**  **“Hello Bob”** |  | | 3 | ***a*: Take hand and shake** | *q*: Acknowledge Bob’s reciprocation. Recognise that Claire is not mutually known | *q*: Realise social exclusion, Desire inclusion | | 4 | *q*: Become acquainted with Claire | ***a*: Introduce Claire**  **“Have you met Claire?”** | *q*: Expect to become acquainted with Alan | | 5 | ***a*: Greet Claire**  **“No. How do you do?”** |  | *q*: Become acquainted with Alan | | 6 | *q*: Accept mutual acquaintance | *q*: Accept mutual acquaintance | ***a*: Respond**  **“Nice to meet you”** |   Note that the events at levels 1 to 4 for agents Alan and Bob form a subsidiary interaction, as do the events at levels 5 and 6 for Alan and Claire. |

*Normative Effects*

Most interactions seem to involve norm-modified behaviour. The example given of friends greeting and introductions being made is a case in point. In that example Adam, Bob, and Claire all accept norms that direct the proper way to behave in the situation in which they find themselves. Moreover, we are often interested in the interactions that are determined by independently proposed or identified collections of rules.

Let *N* be a norm formation, *JS* = (*a1*, …, *an*) an interaction with contexts *CS* = (*cS1*, …, *cSn*), we say that *JS* is an interaction governed in its contexts by *N* and write *g*(*N,* *CS, JS*) iff:

(*i*) (*n*  *N*) [*g*(*n*, *CS***.***i, JS***.***i*)]

» *For every action in the interaction sequence, the action is governed in its context by N, and*

* *CS***.***i* = *cS,i* and *JS***.***i* = *ai*
* Interactions governed by *N*, for whatever *N*, may occur as subsidiary interactions in a larger sequence of actions.

We can include the governing norms conveniently on an action table. For example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *i* = 0 | *x1* | *x2* | *x3* | *x4* |
| 1 | *n*: ***a*** |  | *c* | *c* |
| 2 |  | *q,**c* | *n*: ***a*** |  |
| 3 | *q* | *n*: ***a*** | *c* |  |
| 4 | *c* |  | *n*: ***a*** | *q,**c* |
| 5 | *n*: ***a*** | *c* | *c* | *c* |
| 6 | *q,**c* |  | *q,**c* | *n*: ***a*** |

* The governing norms in any agent history need not include all of *N* – even where *N* is restricted to just the norms required to explain the interaction in question.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Example:**  Consider the following possible revision of the previous example, where we are interested in a norm formation *N* = {*n1*, …, *nm*} proposed to explain behaviour in interpersonal interactions.   |  |  |  |  | | --- | --- | --- | --- | |  | *Alan* | *Bob* | *Claire* | | 1 | ***a*: Enter room** | *q*: Recognise Alan’s presence | *q*: Recognise the presence of an unknown person | | 2 | *q*: Acknowledge Bob’s greeting | *n* **=** Greet friends when they come near after an absence  ***a*: Greet and offer hand**  **“Hello Bob”** |  | | 3 | *n* **=** Reciprocate sincere greetings  ***a*: Take hand and shake** | *q*: Acknowledge Bob’s reciprocation. Recognise that Claire is not mutually known | *q*: Realise social exclusion, Desire inclusion | | 4 | *q*: Become acquainted with Claire | *n* **=** If you are the common acquaintance of two who don’t know each other, make the introduction  ***a*: Introduce Claire**  **“Have you met Claire?”** | *q*: Expect to become acquainted with Alan | | 5 | *n* **=** Reciprocate introductions  ***a*: Greet Claire**  **“No. How do you do?”** |  | *q*: Become acquainted with Alan | | 6 | *q*: Accept mutual acquaintance | *q*: Accept mutual acquaintance | *n* **=** Reciprocate sincere greetings  ***a*: Respond**  **“Nice to meet you”** |   Note that the events at level 1 are not part of the interaction governed by the norms of social gatherings. Event *a1* is not governed by the norm formation in which we are interested. We need not doubt, however, that it is governed by *some* norm formation – such as one which permits one to enter a room if one’s work requires access to equipment therein – but such norms are from a *different* norm formation. |

***Action Networks***

Any relationship between agents *x* and *y* is sociologically relevant just in so far as it implies the possibility of an action by *x* having an act effect on *y* or vice versa. Given the conditions for socially relevant relations it is possible to consider an aggregate of agents as a network of relata. The details of this can be found in texts treating of Social Network Analysis (e.g. Scott, J. & P. Harrington (eds) (2011) *The SAGE Handbook of Social Network Analysis*, London:SAGE; Wasserman, S. & K. Faust (1994) *Social Network Analysis*, Cambridge, UK:CUP; Wellman, B. & S. D. Berkowitz (1988) *Social Structures: A Network Approach*, Cambridge, UK:CUP.) Note that those texts take the conditions for the social relevance of relations to be unproblematic. In the definitions which follow the intention is that the conditions under which the relations may be said to exist, or under which they are to be considered effective are more or less precisely laid out.relations are

A different form of notation, inspired by graph theory, is appropriate for a focus on relations.

*Action Dyads*

Define *x*, *y* as a binary relation indicating the possibility of an action by *x* which has an act effect on *y*. ie. *x*, *y* iff (*ax*  *x*) [*e*(*ax*, *y*)].

* Read this as *x* Acts On *y*
* We also have *x*  *y* as an alternative notation. This is the sort of notation that is commonly used in graph theory. The action element definition allows us to have a full understanding of what is implied by the connecting ‘arrow’ in terms of the agent model sketched above.
* If *x*, *y* and *y*, *x* then *x*  *y* or *x*  *y*, but we don’t need to develop this notation further.

Write *x*, *y**N* iff (*cx,0*  *x*) [*g*(*N*, *cx,0*) & (*ax*  *x*) [(*N,cx,0*(*ax*)  *N,cx,0* (*ax*)) & *e*(*ax*, *y*)]]

» *There is a context governed by the norm formation N, and there is an action by x commanded or permitted by N in that context that has an act effect on y.*

* Read this as *x* Acts On *y* Under *N*
* *N*: *x*  *y*, or, if the action element is part of a graphical representation of a collection of actions all governed by the same norm formation, then let *N* be a label for that representation.
* *x*, *y**N**x*, *y*

Let *A*(x*, y*) = {*ax*  *x*: *e*(*ax*, *y*)}. Then let *p**x*, *y* indicate that there is an objective probability *p* that *x* produces an action that has an act effect on *y*, thus:

*p**x*, *y* iff *x*, *y* and *Prob*(*Ax*(*qx,0*, *cx*,*0*)*A*(x*, y*)) = *p*.

* Read this as *x* Acts On *y* With Probability *p*
* *x* *p* *y*
* Note that this says nothing about the *significance* of the act effect. It is not clear how *significance* can be interpreted, and therefore how it can be modeled.

*p**x*, *y**N* iff *x*, *y**N* and *Prob*(*g*(*N*, *cx,0*) & *Ax*(*qx,0*, *cx*,*0*)=*ax*  (*ax**A*(x*, y*)) & (*N,cx,0*(*ax*)*N,cx,0*(*ax*)))) = *p*

* Read this as *x* Acts On *y* Under *N* With Probability *p*
* *N*: *x* *p* *y*

*Dyadic Action Diagrams*

Let *X* be a set of agents, *t*  [0, 1], *A*(x*, y*) = {*ax*  *x*): *e*(*ax*, *y*)}as before.

Define *Y*(*X*2) = {(*x*, *y*): *x, y*  *X*, and *x*, *y*}

*Z*(*X*2) = &(*x*,*y*)*Y*(*X*2) *x*, *y* is the Action Diagram for *X*

Define *Y*(*X*2, *t*) = {(*x*, *y*): *x, y*  *X*,*x*, *y*, and *Prob*(*Ax*(*qx,0*, *cx*,*0*)*A*(x*, y*))  *t*}; then

*Z*(*X*2, *t*) = &(*x*,*y*)*Y*(*X*2,*t*) *x*, *y* is the Action Diagram for *X* With Probability  *t*

* *t* is the Diagram Threshold Probability whose significance is only to distinguish a level of probable action below which the actions are not considered structurally significant.

Define *YN*(*X*2) = {(*x*, *y*): *x, y*  *X*, and*x*, *y**N*}; then

*ZN*(*X*2) = &(*x*,*y*)*YN*(*X*2) *x*, *y**N* is the Action Diagram for *X* Under *N*

Define *YN*(*X*2, *t*) = {(*x*, *y*): *x, y*  *X*,*x*, *y**N*,   
and *Prob*(*g*(*N*, *cx,0*) & *Ax*(*qx,0*, *cx*,*0*)=*ax*  (*ax**A*(x*, y*)) & (*N,cx,0* (*ax*)  *N,cx,0* (*ax*))))  *t*}; then

*ZN*(*X*2, *t*) = &(*x*,*y*)*YN* (*X*2,*t*) *x*, *y**N*, for *N* a norm formation, is the Action Diagram for *X* Under *N* With Probability  *t*

*Dyadic Action Relations*

Let *r* be a relation defined on *X*2

When (*x,y*)[*r*(*x*, *y*)*x*, *y*] then *x* Acts On *y* in *r*

* Only in this case is the relationship *r* sociologically Relevant, and we write *R*(*r*)
* We also have *r*: *x*  *y* as an alternative notation for *r*(*x* *y*), which indicates that the action element *x*  is implied by the relation *r*; or, if the action element is part of a graphical representation of a collection of action elements all implied by the same relation, then let *r* be a label for that representation.
* *Yr*(*X*2) = {(*x*, *y*): *r*(*x*, *y*) and (*x*, *y*)  *Y*(*X*2)}
* *Zr*(*X*2) = &(*x*,*y*)*Yr*(*X2*) *r*(*x*, *y*) is the Action Diagram for *r* In *X*

When (*x,y*)[*r*(*x*, *y*)*x*, *y**N*] then *x* Acts On *y* In *r* Under *N*

* Note that (*x*,*y*)[*r*(*x*, *y*)*N**x*, *y**N*]](*x,y*)[*r*(*x*, *y*)*x*, *y*]  *R*(*r*)
* *r*: *N*: *x*  *y*
* *YN,r*(*X*2) = {(*x*, *y*): *r*(*x*, *y*), (*x*, *y*)  *x*, *y**N*}
* *ZN,r*(*X*2) = &(*x*,*y*)*YN,r*(*X2*) *r*(*x*, *y*) is the Action Diagram for *r* In *X* Under *N*

When (*x,y*)[*r*(*x*, *y*)*p**x*, *y*] then *x* Acts On *y* In *r* With Probability *p*

* Note that (*x*,*y*)[*r*(*x*, *y*)*p**x*, *y*](*x,y*)[*r*(*x*, *y*)*x*, *y*]  *R*(*r*)
* *r*: *x*  *p* *y*
* *Yr*(*X*2, *t*) = {(*x*, *y*): *r*(*x*, *y*) and (*x*, *y*)  *Y*(*X*2, *t*)}
* *Zr*(*X*2, *t*) = &(*x*,*y*)*Yr*(*X*2,*t*) *x*, *y* is the Action Diagram for *r* In *X* With Probability  *t*

When (*x,y*)[*r*(*x*, *y*)*p**x*, *y**N*] then *x* Acts On *y* In *r* Under *N* With Probability *p*

* Note that (*x*,*y*)[*r*(*x*, *y*)*p**x*, *y**N*](*x,y*)[*r*(*x*, *y*)*x*, *y*]  *R*(*r*)
* *r*: *N*: *x*  *p* *y*
* *YN,r*(*X*2, *t*) = {(*x*, *y*): *r*(*x*, *y*) and (*x*, *y*)  *YN*(*X*2, *t*)}
* *ZN,r*(*X*2, *t*) = &(*x*,*y*)*YN,r*(*X*2,*t*) *x*, *y**N*, for *N* a norm formation, is the Action Diagram for *X* Under *N* With Probability  *t*

|  |
| --- |
| **Example:**  Consider the relation described by ‘*x* hates *y.*’ This is a sociologically relevant relationship because it is in the nature of hatred that if possible the hater will do what it can to damage the object of that hatred. Thus ‘*x* hates *y*’*x*, *y*.  Consider also the relation described by ‘*x* is the brother of *y*.’ This is a sociologically relevant relationship because there is a norm formation *Nfraternality* of action-directing rules that describes the range of actions that are expected of persons in this relationship. In that case we say that ‘*x* is the brother of *y*’*x*, *y**Nfraternality*. If such rules do not exist then the relationship is sociologically inert – like the relationships ‘*x* is the unwitting brother of *y*’ or ‘*x* has the same birthday as *y*.’ |